Koyana Education Society's

Balasaheb Desai College, Patan

Teaching Plan

2023-2024

Subject Name – Mathematics **Class-**B.Sc. III

Paper Name –Linear Algebra **Month –**February

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01/02/2024	#Linear Span - Definition of Linear Span and Examples. Theorem: (S) is the smallest subspace of V containing S. Theorem: If $S1$ and $S2$ are subsets of V, then (i) $S1 \subseteq S2 \Rightarrow L(S1) \subseteq L(S2)$ (ii) $L(S1 \cup S2) = L(S1) + L(S2)$ (iii) $L(L(S1)) = L(S1)$	Lecture and Problem solving
03/02/2024	Theorem: If W is subspace of V then $(W) = W$ and conversely. Definition of Finite dimensional vector space (F.D.V.S). # Linear dependence, independence and basis. Definition: Linear dependence (L. D.) and independence (L. I.), basis of vector space.	Lecture and Problem solving
05/02/2024	Examples of Linear dependence, independence and basis. Theorem: If $S = \{v_1, v_2, v_3, . .. , v_n\}$ is a basis of V then every element of V can be expressed uniquely as a linear combination of $v_1, v_2, v_3, . .. , v_n$. Theorem: Suppose S is a finite subset of a vector space V such that $V = (S)$ then there exists a subset of S which is a basis of V. Corollary: A F.D.V.S. has a basis	Lecture and Problem solving
06/02/2024	Theorem: Let V be a F.D.V.S. Suppose S and T are two finite subsets of V such that S spans V and T is L. I. Then $(T) \leq (S)$. Corollary : Any basis of F.D.V.S. V is finite. Corollary : Any two bases of a F.D.V.S. have same number of elements. Definition of dimension.	Lecture
07/02/2024	Corollary : If dim $V = n$, then any $n + 1$ vectors in V are linearly dependent. Theorem: (without proof) A basis of a vector space is a maximal linearly independent set and conversely every maximal linearly independent set in a vector space is its basis. Corollary: (without proof) Suppose n is the maximum number of L. I. vectors in any	Lecture and Problem solving

	subset of a vector space V. Then dim $V = n$. Theorem: (without proof) Let (F) be a vector space. A minimal generating set of V is a basis of V and conversely, every basis of V is a maximal generating set. Theorem: If V is a F.D.V.S. and $\{v_1, v_2, v_3, \ldots, v_r\}$ is a L.I. subset of V, then it can be extended to form a basis of V.	
09/02/2024	Theorem: If dim $V = n$ and $S = \{v_1, v_2, v_3, \ldots, v_n\}$ spans V, then S is a basis of V Theorem: If dim $V = n$ and $S = \{v_1, v_2, v_3, \ldots, v_n\}$ is a L.I. subset of V, then S is a basis of V.. Theorem: (without proof) Two finite dimensional vector spaces over F are isomorphic if and only if they have the same dimension.. Thorem: (without proof) Let W be a subspace of a F.D.V.S. V, then W is finite dimensional and $dim\ W \le dim\ V$. In fact, $dim\ V = dim\ W$ iff $V = W$.	Lecture and Problem solving
10/02/2024	#Properties of Linear Transformation. Theorem: A L.T. $T : V \to V$ is one – one iff T is onto. Theorem: Let V and W be two vector spaces over F. Let $\{v_1, v_2, \ldots, v_r\}$ be a basis of V and $w_1, w_2, \ldots, w_n$ be any vectors in W (not essentially distinct). Then there exists a unique L.T. $T: V \to W$ such that ( ) = w ; $i = 1, 2, \ldots, n$ Definition of rank and nullity of a linear transformation. Theorem: (Sylvester's Law) Let $T: V \to W$ be a linear transformation. Then $Rank\ T + Nullity\ T = dim\ V$. Theorem: If $T : V \to V$ be a L.T., then the following statements are equivalent. i) $Range\ T \cap Ker\ T = \{0\}$ ii) If $((v)) = 0$ then $(v) = 0$, $v \in V$.	Lecture and Problem solving
11/02/2024	# Algebra of Linear Transformations. Definition of Sum and scalar multiple of L.T. The vector space Hom (V, W). Definition of Product (composition) of L.T. s, Linear operator, Linear functional. Theorem: Let T, $T1$, $T2$, $T3$ be linear operators on V and let $I: V \to V$ be the identity mapping $I(v) = v$ for all v then i) $I\ T = T\ I = T$ ii) $T(T1 + T2) = TT1 + TT2$ $(T1 + T2)T = T1T + T2T$ iii) $\alpha(T1\ T2) = (\alpha\ T1)\ T2 = T1\ (\alpha\ T2)$; $\alpha \in F$ iv) $T1\ (T2\ T3) = (T1\ T2)\ T3$	Lecture
12/02/2024	Theorem: (without proof) Let V and W be two vector spaces (over F) of dimension m and n respectively. Then Hom (V, W) has dimension mn. # Invertible Linear	Lecture and Problem solving

	Transformation. Definition of Invertible map, Inverse of a L.T. is also a L. T., Definition of nonsingular L. T. Theorem: A L.T. $T : V \to W$ is a non-singular iff T carries each L. I. subset of V onto a L. I. subset of W	
13/02/2024	Theorem: Let $T : V \to W$ be a L.T. where V and W are F.D.V.S. with same dimension. Then the following statements are equivalent. (i) T is invertible. (ii) T is non-singular. (iii) T is onto. (iv) If $\{v1, v2, \ldots, vn\}$ is a basis of Vthen $\{T(v1), T(v2), \ldots, T(vn)\}$ is a basis of W.Theorem: Let $T : V \to W$ and $S : W \to U$ be two linear transformations. Then (i) If S and T are one – one, onto then ST is one-one, onto and $(ST) -1 = T -1S -1$ (ii) If ST is one – one then T is one-one. (iii) If ST is onto then S is onto.	Lecture and Problem solving
14/02/2024	Matrix of L.T. and examples. Definition of Matrix of L.T. and examples. Theorem: $Hom\ (U, V) \cong Mm\times n$ (�)	Lecture and Problem solving
15/02/2024	Unit 2: Inner Product Spaces, Eigen values and Eigen vectors. # Inner product space . Definition of Inner product space, norm of a vector and examples	Lecture
16/02/2024	Theorem: Cauchy- Schwarz inequality. Let V be an inner product space. Then $\|(u, v)\| \leq \|u\| \|v\|$, for all $u, v \in V$.Theorem: Triangle inequality. Let V be an inner product space. Then $\|x + y\| \leq \|x\| + \|y\|$, for all $x, y \in V$. AND EXAMPLES	Lecture and Problem solving
17/02/2024	Theorem: Cauchy- Schwarz inequality. Let V be an inner product space. Then $\|(u, v)\| \leq \|u\| \|v\|$, for all $u, v \in V$. Theorem: Triangle inequality. Let V be an inner product space. Then $\|x + y\| \leq \|x\| + \|y\|$, for all $x, y \in V$.AND EXAMPLES	Lecture and Problem solving
21/02/2024	Theorem: Generalized Pythagoras Theorem Let V be an inner product space. Let $x, y \in V$ such that $x \perp y$. Then $\|x + y\|^2 = \|x\|^2 + \|y\|^2$. Definition of orthonormal set. Theorem: Let S be a orthogonal set of non-zero vectors in an inner product space V. Then S is a linearly independent set.AND EXAMPLES	Lecture and Problem solving

22/02/2024	Corollary: An orhtonormal set in an inner product space is L. I. Theorem: (Gram-Schmidt orthogonalisation process) Let V be a non-zero inner product space of dimension n. Then V has an orthonormal basis	Lecture
23/02/2024	Examples on Gram-Schmidt orthogonalisation process (finding the orhtonormal basis). Theorem: Bessel's inequality. If $\{w_1, w_2, \ldots, w\}$ be an orthonormal set in V, then $\sum_{i=1}^{m}|(w_i, v)|^2 \le \|v\|^2$ for all $u, v \in V$. Eigen values and Eigen vectors . Definition of Eigen values, Eigen vectors and simple examples.	Lecture and Problem solving
24/02/2024	Definition of Eigen space of T associated with Eigen value. Eigen space is a subspace. Theorem: Let T be a linear operator on a finite dimensional vector space V over F. Then $c \in F$ is an Eigen value of T if and only if $T - cI$ is singular (not invertible). Property 1: Let dim $V = n$. Let T be a linear operator on V. Let $v_1, v_2, \ldots, v_k$ be Eigen vectors of T, corresponding to distinct Eigen values c_1, $c_2, \ldots, c_k$ of T. Then $v_1, v_2, \ldots, v_k$ are linearly independent.	Lecture and Problem solving
25/02/2024	Charateristic Polynomials - Definition of Characteristic Polynomial of a matrix and remarks on it. Definition of similar matrices. Theorem: Similar matrices have same characteristic polynomial	Lecture
26/02/2024	Definition of Characteristic Polynomial of a Linear operator. Theorem: Let $c \neq 0$ be an Eigen value of an invertible operator T. Then c^{-1} is an Eigen value of T^{-1}	Lecture
27/02/2024	Examples on Eigen values and Eigen vectors, real life application (Predatory – Prey problem)	Lecture and Problem solving
28/02/2024	Examples	Problem solving

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