Shivaji University, Kolhapur

B.Sc. III (CBCS) Mathematics

Semester V - Paper XI

Subject Code: 79674

Optimization Technique

Question Bank

Q1. Choose Correct Alternative. (1 marks each)

1. The objective of sequencing problem is --------.
   A) To find the order in which jobs are to be made.
   B) To find the time required for the completing all the job on hand.
   C) To find the sequence in which jobs on hand are to be processed to minimize the total time required for processing the jobs.
   D) To maximize the cost
2. In a 3 machine and 5 jobs problem, the least of processing times on machine A, B and C are 5, 1 and 3 hours and the highest processing times 9, 5 and 7 respectively, then it can be converted to a 2-machine problem if order of the machine is----------.
   A) B-A-C
   B) A-B-C
   C) C-B-A
   D) Both (B) and (C)
3. In order for a transportation matrix which has five rows and four columns not to be degenerate, how much must be the number of allocated cells in the matrix?
   A) 9
   B) 7
   C) 8
   D) 20
4. Calculating cell evaluations (unit cost differences) d_{ij} for each empty cell (i,j) by using the formula $d_{ij} = c_{ij} - (u_i + v_j)$ is one of the steps of which method?
   A) VAM
   B) Lowest cost entry method
   C) MODI method
   D) Hungarian method

5. An assignment problem is considered as a particular case of a transportation problem because_____.
   A) The number of rows equals columns
   B) All x_{ij} =0 or 1

C) All rim conditions are 1
D) All of the above

6. The modified distribution (MODI) method is also known as _____.
   A) U-V method or method of multipliers
   B) Stepping stone method
   C) Matrix minima method
   D) Unit cost penalty method
7. For the game with pay off matrix :

		Player B		
		B_1	B_2	B_3
Player A	A_1	-1	2	-2
	A_2	6	4	-6

The game is _____.

   A) Fair game
   B) Strictly determinable
   C) Not strictly determinable
   D) None of these

8. Total elapsed time to process all jobs through two machines is given by_____.
   A) $\sum_{j=1}^{n} M_{2j} + \sum_{j=1}^{n} M_{1j}$
   B) $\sum_{j=1}^{n} M_{2j} - \sum_{j=1}^{n} M_{1j}$
   C) $\sum_{j=1}^{n} M_{1j} + \sum_{j=1}^{n} I_{1j}$
   D) None of the above
9. The feasible region represented by the constraint $x_1 + x_2 \leq 1, 3x_1 + x_2 \geq 3,$ $x_1 \geq 0,\ x_2 \geq 0$ of the objective function $z = x_1 + 2x_2$ is_____.
   A) A Polygon
   B) Empty set
   C) Unbounded set
   D) A singleton set
10. In sequencing an optimal path is one that minimizes ----------.
    A) Elapsed time
    B) Idle time
    C) Both A) and B)
    D) Ready time
11. For the LP problem: Maximize $z = 3x + 2y$ subject to, i) $x - y \geq 1$, ii) $x + y \geq 3$ $x, y \geq 0$
    Which of the following is true?
    A) No feasible solution exists.
    B) Bounded feasible solution exists.
    C) Unbounded feasible solution exists.

D) None of these

12. A game is said to be fair if_____.
    A) Upper value is more than lower value of the game.
    B) Upper and lower values of the game are not equal.
    C) Both upper and lower values of the game are same and zero.
    D) None of the values
13. The value of the following 2 x 2 game without saddle point using arithmetic method is_____.

		Player B	
		B_1	B_2
Player A	A_1	5	1
	A_2	3	4

    A) 17/5
    B) 5/17
    C) 7/15
    D) 7/5

14. To convert unbalanced transportation problem with total supply equals to 40 & total demand equals to 50 into balanced problem we add_____ .
    A) Dummy column with demand 10
    B) Dummy column with demand 20
    C) Dummy row with supply 10
    D) Dummy row with supply 20
15. Necessary and sufficient condition for existence of a feasible solution to $m \times n$ transportation problem is_____.
    A) $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$
    B) $\sum_{i=1}^{m} a_i > \sum_{j=1}^{n} b_j$
    C) $\sum_{i=1}^{m} a_i < \sum_{j=1}^{n} b_j$
    D) $\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j$
16. An optimization model _____.
    A) Mathematically provides the best decision
    B) Provides decision within its limited context
    C) Helps in evaluating various alternatives constantly
    D) All of the above
17. A firm manufactures two type of products A and B sells them at a profit of Rs 4 on type A and Rs 3 on type B. Each product is processed on two machines G and H. Type A requires 1 minute of processing time on G and 2 minutes on H. Type B requires 1 minute on G and 1 minute on H. The machine G available for not more than 6 hour while machine H is available for 10 hours during any working day. What is the objective function?

A) $z = 4x + 3y$
B) $z = x + y$
C) $z = x + 2y$
D) $z = 6x + 10y$

18. Maximum assignment problem is transformed into a minimum problem by -----.
    A) Adding each entry in a column from the maximum value in that column.
    B) Subtracting each entry in the table from the maximum in that table.
    C) Subtracting each entry in a column from the maximum value in that column
    D) None of the above.

19. In marking assignments, which of the following should be preferred?
    A) Only row having single zero
    B) Only column having single zero
    C) Only row/column having single zero
    D) Column having more than one zero

20. In a 7×7 transportation problem, degeneracy would arise if the number of filled slots were _____.
    A) equal to 49
    B) equal to 14
    C) more than 14
    D) less than 13

21. The mathematical model of an LP problem is important because------.
    A) It helps in converting the verbal description & numerical data into mathematical expression
    B) It captures the relevant relationship among decision
    C) Decision-makers prefer to work with formal models
    D) It enables the use of algebraic technique

22. Which statement is true about the game $\begin{bmatrix} 1 & -3 \\ 4 & 1 \end{bmatrix}$?
    A) game is fair.
    B) value of the game is 4.
    C) value of the game is 1.
    D) no saddle point exists.

23. An optimization model _____.
    A) Mathematically provides the best decision
    B) Provides decision within its limited context
    C) Helps in evaluating various alternatives constantly
    D) All of the above

24. The time during which a machine remains waiting or vacant in sequencing problem is called time.
    A) Processing

B) Waiting
C) Free
D) Idle

25. The positive variable which is added to left hand side of the constraints, so as to bring them into equality are called as _____.
    A) Slack variables
    B) Surplus variables
    C) Artificial variables
    D) None of these

26. An assignment problem with 3 rows & 5 columns is converted into balanced assignment problem by _____.
    A) adding 2 columns with each cost 0
    B) adding 2 rows with each cost 0
    C) adding 1 columns with each cost 0
    D) adding 3 rows with each cost 0

27. If there are n jobs and m machines then there will be ---------- sequences of doing the jobs.
    A) mn B) m(n!) C) n^m D) $(n!)^m$

28. Every basic feasible solution of a general assignment problem, having a square payoff matrix of order n should have assignments equal to _____.
    A) 2n+1
    B) m+n
    C) m+n-1
    D) 2n-1

29. Total time required to complete all the jobs in a job sequencing problem is known as ----------.
    A) processing time
    B) waiting time
    C) elapsed time
    D) idle time

30. The initial solution of a T.P. obtained by _____.
    A) North-West Corner Rule would in variably be optimum
    B) Least cost method does not provide that least cost solution to a T.P.
    C) VAM would invariably be very near to optimum solution
    D) MODI method is infeasible

31. The L.P.P. Min $z = -x + 2y$ subject to $-x + 3y \leq 10, x + y \leq 6, x - y \leq 2, x \geq 0, y \geq 0$ which of the following coordinate is corner point of the region of the feasible solutions of above L.P.P. ?
    A) (0,0)
    B) (4,2)
    C) (2,5)
    D) (1,2)

32. In solving 2 machine and n jobs sequencing problem, the following assumption is wrong.
    A) No passing is allowed
    B) Processing times are known
    C) Handling time is negligible
    D) The time of passing depends on the order of machining.
33. The purpose of a dummy row or column in an assignment problem is to
    A) Obtain balance between total activities &total resources
    B) Prevent a solution from becoming degenerate
    C) Provide a means of representing a dummy problem
    D) None of the above
34. The method of finding an initial solution based upon opportunity costs is called _____.
    A) the northwest corner rule
    B) Vogel's approximation
    C) Flood's technique
    D) Hungarian method
35. The value of the game in general satisfies the equation_____.
    A) maxmin value $\geq V \leq$ minmax value
    B) maxmin value $= V =$ minmax value
    C) minmax value $\leq V \leq$ maximin value
    D) maximin value $\leq V \leq$ minmax value
36. An assignment problem is considered as a particular case of a transportation problem because_____.
    E) The number of rows equals columns
    F) All x_{ij} =0 or 1
    G) All rim conditions are 1
    H) All of the above
37. In a traveling salesman problem, the elements of diagonal from left-top to right bottom are
    A) Zeros
    B) All negative elements
    C) All ones
    D) All infinity
38. While solving a LP model graphically, the area bounded by the constraints is called_____.
    A) Feasible region
    B) Infeasible region
    C) Empty region
    D) None of the above
39. The method of finding an initial solution based upon opportunity costs is called _____.
    A) the northwest corner rule
    B) Vogel's approximation

C) Flood's technique
D) Hungarian method

40. For the following game:

		Player B		
		I	II	III
Player A	I	3	-4	8
	II	-8	5	-6
	III	6	-7	6

Which of the following is true?
A) Only one saddle point exists
B) Two saddle points exists
C) No saddle point exists
D) Three saddle points exists

Q2. Long answer type questions (8 marks each)

1. Explain North – West rule to find an initial basic feasible solution of a transportation problem and hence obtain the initial basic feasible solution of the following transportation problem by North-West corner rule

	A	B	C	D	E	Supply
M_1	2	11	10	3	7	4
M_2	1	4	7	2	1	8
M_3	3	9	4	8	12	9
Demand	3	3	4	5	6	

2. Explain Low cost entry method to find an initial basic feasible solution of a transportation problem and hence obtain the initial basic feasible solution of the following transportation problem by Low cost entry method

	A	B	C	Supply
M_1	2	7	4	5
M_2	3	3	1	8
M_3	5	4	7	7
M_4	1	6	2	14
Demand	7	9	18	

3. Prove that the number of basic variables in a transportation problem are at the most m+n-1.
4. What is an assignment problem? Explain Hungarian method to solve assignment problem.
5. Use the graphical method to solve the following LP problem
   Maximize $z = 6x - 4y$, subject to the constraints
   $2x + 4y \leq 4\,,\ 4x + 8y \geq 16\,,\ x, y \geq 0.$
6. Use the graphical method to solve the following LP problem
   Maximize $z = 5x + 3y$, subject to the constraints
   $3x + 5y \leq 15,\ 5x + 2y \leq 10\,,\ x, y \geq 0.$
7. Explain transportation algorithm for minimization problem (MODI Method).
8. Determine the initial basic feasible solution of the following transportation problem by Vogel's approximation method and test it for optimality.

Factories

Warehouse	D1	D2	D3	D4	Supply
O1	19	30	50	10	7
O2	70	30	40	60	9
O3	40	5	70	20	18
Demand	5	8	7	14	

9. Determine the initial basic feasible solution of the following transportation problem by Vogel's approximation method and test it for optimality. Find the optimal schedule and minimum transportation cost.

Destinations

Origins	D1	D2	D3	D4	Supply
O1	1	2	1	4	30
O2	3	3	2	1	50
O3	4	2	5	9	20
Demand	20	40	30	10	

10. Determine the initial basic feasible solution of the following transportation problem by Vogel's approximation method and test it for optimality.

Destinations

Origins	D1	D2	D3	D4	Supply
O1	11	13	17	14	250
O2	16	18	14	10	300
O3	21	24	13	10	400
Demand	200	225	275	250	

11. Determine the initial basic feasible solution of the following transportation problem by Vogel's approximation method and test it for optimality.

Destinations

Origins		D1	D2	D3	D4	D5	Supply
	O1	2	11	10	3	7	10
	O2	1	4	7	2	1	2
	O3	3	9	4	8	12	9
	Demand	3	3	4	5	6	

12. Solve the following game by graphical method.

		Player B			
		I	II	III	IV
Player A	I	2	1	0	-2
	II	1	0	3	3

13. Solve the following game by graphical method.

		Player B			
		I	II	III	IV
Player A	I	1	4	-2	-3
	II	2	1	4	5

14. Solve the following game by graphical method.

		Player B			
		I	II	III	IV
Player A	I	8	5	-7	9
	II	-6	6	4	-2

15. A city corporation has decided to carry out road repairs on main four arteries of the city. The government has agreed to make special grant of Rs. 50 lakhs towards the cost with a condition that the repairs must be done at the lowest cost and in quickest time. If conditions warrant, then a supplementary token grant will be also be considered favourably. The corporation has floated tenders and 5 contractors have sent in their bids. In order to expedite work, one road will be awarded to only one contractor.

Cost repairs Lakhs

Road	D1	D2	D3	D4
O1	9	14	19	15
O2	7	17	20	19
O3	9	18	21	18
O4	10	12	18	19

O5	10	15	21	16

i) Find the best way of assigning the repair work to the contractors and the costs.

ii) If it is necessary to seek supplementary grants then what should be the amount sought?

Q3. Short answer type questions (4 marks each)

1. Solve the game whose payoff matrix is given by

		Player B		
		I	II	III
Player A	I	-2	15	-2
	II	-5	-6	-4
	III	-5	20	-8

2. A book company has one printing machine and one binding machine there are numbers of different books processing time for printing and binding are given below

Books	A	B	C	D	E
Printing time	5	1	9	3	10
Binding time	2	6	7	8	4

3. Find the sequence that minimizes the total elapsed time required to complete the following jobs:

	Processing times in hours					
Jobs:	1	2	3	4	5	6
Machine A	4	8	3	6	7	5
Machine B	6	3	7	2	8	4

4. We have 6 jobs, each of which must go through machines A, B and C in the order A,B,C. Processing time in hours are given in the following table:

	Processing times in hours					
Jobs:	1	2	3	4	5	6
Machine A	8	3	7	2	5	1
Machine B	3	4	5	2	1	6
Machine C	8	7	6	9	10	9

Determine a sequence for the jobs that will minimize the total elapsed time.

5. Give Johnson's procedure for determining an optimal sequence for processing n items on two machines.

6. Prove that if mixed strategies are allowed then there always exists a value of the game.
7. Define assignment problem and give the mathematical formulation of it.

8. For what value of λ the game with the following payoff matrix is strictly determinable?

		Player B		
		I	II	III
Player A	I	λ	6	2
	II	-1	λ	-7
	III	-2	4	λ

9. Solve the following game by arithmetic method.

		Player B		
		I	II	III
Player A	I	1	7	2
	II	6	2	7
	III	5	1	6

10. Solve the following game algebraic method.

		Player B		
		I	II	III
Player A	I	-1	-2	8
	II	7	5	-1
	III	6	0	12

11. Solve the assignment problem.

		Projects			
		P1	P2	P3	P4
Group	G1	20	22	28	15
	G2	16	20	12	13
	G3	19	23	14	25
	G4	10	16	12	10

12. Solve the assignment problem.

		Machines			
		A	B	C	D
Jobs	1	10	12	19	11
	2	5	10	7	8
	3	12	14	13	11
	4	8	15	11	9

13. Find the optimal assignment for the given assignment

Jobs \ Machines	1	2	3
1	5	7	9
2	14	10	12
3	15	13	16

14. State the travelling salesman problem and formulate it as an assignment problem.
15. Explain Vogel's approximation method.
16. Explain Matrix-Minima Method.
17. Discuss the algorithm of stepping stone method.
18. Prove that a necessary and sufficient condition for the existence of feasible solution of a transportation problem is $\sum a_i = \sum b_j$, $(i = 1,2, \dots, m; j = 1,2, \dots, n)$.
19. Define the following terms:
    i) Feasible solution
    ii) Basic Feasible solution
    iii) Optimum Basic Feasible solution
    iv) Unbounded Solution
20. Obtain an initial basic feasible solution to the following Transportation Problem using the matrix minima method.

Warehouse \ Factories	D1	D2	D3	D4	Supply
O1	1	2	3	4	6
O2	4	3	2	0	8
O3	0	2	2	1	10
Demand	4	6	8	6	

21. Find an initial basic feasible solution to the following Transportation Problem using the stepping stone method.

Origin \ Destination	D1	D2	D3	D4	Supply
O1	1	5	3	3	34
O2	3	3	1	2	15
O3	0	2	2	3	12
O4	2	7	2	4	19
Demand	21	25	17	17	

22. A machine operator has to perform two operations, turning and threading, on a number of different jobs. The time required to perform these operations (in minutes) for each job is known as below:

Job	Time of turning (minutes)	Time of threading (minutes)
1	3	8
2	12	10
3	5	9
4	2	6
5	9	3
6	11	1

Find the sequence that minimizes the total elapsed time required to complete all the jobs.

23. Obtain an initial basic feasible solution to the following Transportation Problem using the matrix minima method.

Factories

Warehouse	D1	D2	D3	D4	Supply
O1	6	3	5	4	22
O2	5	9	2	7	15
O3	5	7	8	6	8
Demand	7	12	17	9	

24. Find an initial basic feasible solution to the following Transportation Problem using the stepping stone method.

Destination

Origin	D1	D2	D3	D4	Supply
O1	6	4	1	5	14
O2	8	9	2	7	16
O3	4	3	6	2	5
Demand	6	10	15	4	35

25. Solve the following (2 x 2) game without saddle point by arithmetic method.

		Player B	
		B1	B2
Player A	A1	2	-1
	A2	-1	0

26. Solve the following game algebraic method.

		Player B		
		I	II	III
Player A	I	50	40	28
	II	70	50	45
	III	75	47.5	50

27. A salesman has to visit 5 cities A, B , C , D, E. The distance (in 100 kms) between 5 cities are as follows. If the salesman starts from city A, which route he should select so that the total distance travelled is minimum?

	A	B	C	D	E
A	–	1	6	8	4
B	7	–	8	5	6
C	6	8	–	9	7
D	8	5	9	–	8
E	4	6	7	8	–

28. Given the matrix of set up costs, show how to sequence the production so as to minimize the set-up cost per cycle.

	A	B	C	D	E
A	∞	2	5	7	1
B	6	∞	3	8	2
C	8	7	∞	4	7
D	12	4	6	∞	5
E	1	3	2	8	∞

29. Define linear programming problem. A firm manufactures 3 products A, B and C. The profits are Rs. 3, Rs. 2 and Rs. 4 respectively. The firm has two machines and below is the required processing time for each machine on each product.

		Product		
		A	B	C
Machine	G	4	3	5
	H	2	2	4

Machine G and H have 2000 and 5000. The firm must manufacture 100 A's, 200 B's and 50 C's, but no more than 150 A's. Set up L. P. Problem to maximize profit.

30. A firm can produce three types of cloths, say: A,B,C. Three kinds of wool are required for it : red, green and blue wool. One unit length of type A cloth needs 2 meters of red wool and 3 meters of blue wool; One unit length of type B cloth needs 3 meters of red wool , 2 meters of blue wool and 2 meters of green wool; One unit length of type C cloth needs 5 meters of green wool and 8 meters of blue wool. The firm has only a stock of 8 meters of red wool, 10 meters of green wool and 15 meters of blue wool. It is assumed that the income obtained from one unit length of type A cloth is Rs. 3.00, of type B cloth is Rs. 5.00 and of type C cloth is Rs. 4.00. Formulate L. P. Problem to maximize income with available material.