

Shivaji University, Kolhapur
Question Bank For
B.Sc. (Part –III) (Semester –VI) Examination, March/April-2022
MATHEMATICS
Complex Analysis (Paper-XV) Sub. Code: 81664

Multiple Choice Questions

- 1 Functions satisfying Laplace equation are known as.....
 - a) Homomorphic
 - b) conjugate
 - c) harmonic
 - d) regular
- 2 If $f(z)$ is analytic and equals $u(x, y) + iv(x, y)$ then $f'(z) =$
 - a) $\frac{\partial u}{\partial x} - i \frac{\partial u}{\partial y}$
 - b) $\frac{\partial v}{\partial y} - i \frac{\partial v}{\partial x}$
 - c) $\frac{\partial u}{\partial x} - i \frac{\partial v}{\partial x}$
 - d) $\frac{\partial v}{\partial y} + i \frac{\partial v}{\partial x}$
- 3 An analytic function with constant modulus is
 - a) variable
 - b) zero
 - c) does not exist
 - d) constant
- 4 Let $f(z) = |z|^2$ then which of the following statement is correct
 - a) is continuous everywhere
 - b) f is differentiable at $z \neq 0$

- c) f is not analytic at $z = 0$
- d) is not continuous everywhere

5 The value of m so that $2x - x^2 + my^2$ may be harmonic is

- a) 0
- b) 1
- c) 2
- d) 3

6 The value of integral $\int_0^{1+i} z dz$ is

- a) 0
- b) i
- c) $1 + 2i$
- d) $1 - 2i$

7 Harmonic conjugate of $u(x, y) = e^x \cos y$ is

- a) $e^y \cos x$
- b) $e^x \cos y$
- c) $e^x \sin y + c$
- d) $\frac{e^y}{\sin x}$

8 The value of contour integral $\int_c \frac{z^3}{(z-2)^2} dz$ over $|z| = 1$ is

- a) $24\pi i$
- b) $12\pi i$
- c) $6\pi i$
- d) 0

9 Polar form of Cauchy Riemann equations are.

- a) $\frac{\partial u}{\partial r} = \frac{1}{r} \frac{\partial v}{\partial \theta}, \frac{\partial v}{\partial r} = \frac{-1}{r} \frac{\partial u}{\partial \theta}$
- b) $\frac{\partial v}{\partial \theta} = \frac{1}{r} \frac{\partial u}{\partial r}, \frac{\partial u}{\partial r} = \frac{-1}{r} \frac{\partial v}{\partial \theta}$
- c) $\frac{\partial u}{\partial r} = r \frac{\partial v}{\partial \theta}, \frac{\partial v}{\partial r} = -r \frac{\partial u}{\partial \theta}$
- d) $\frac{\partial u}{\partial r} = r \frac{\partial v}{\partial \theta}, \frac{\partial v}{\partial r} = \frac{-1}{r} \frac{\partial u}{\partial \theta}$

10 If C is the circle $|z - a| = r$ then $\int_C \frac{dz}{(z-a)^n} = 2\pi i$ when,..

- a) $n = 1$
- b) $n \neq 1$
- c) $n = 0$
- d) $n = \infty$

11 The order of the pole of the function $f(z) = \frac{z^4+2z+1}{z^2+5z+2}$ at $z = \infty$ is

- a) 2
- b) 1
- c) 0
- d) 4

12 The complex function $f(z) = e^z$ at the point $z = \infty$ is

- a) a pole of order 1
- b) a pole of order 2
- c) an isolated essential singularity
- d) a non-isolated essential singularity

13 The value of $\int_C \frac{dz}{z+2}$ where C is $|z| = 1$ is

- a) $2\pi i$
- b) $-2\pi i$
- c) $4\pi i$
- d) 0

14 Residue of $f(z) = \int_C \frac{z^3}{(z-1)^4(z-2)(z-3)}$ at $z = 3$ is

- a) $\frac{101}{16}$
- b) -8
- c) $\frac{27}{16}$
- d) 0

15 If $\omega = u + iv$ be analytic function of $z = x + iy$ then the families of curves $u(x, y) = c_1$ & $v(x, y) = c_2$ form.

- a) Orthogonal system
- b) Conjugate system

- c) Harmonic system
- d) Analytic system

16 The value of the integral $\int_c \frac{dz}{z-a}$ where c is given by the equation $|z - a| = R$ is

- a) $2\pi i$
- b) $\frac{\pi i}{2}$
- c) $-\pi i$
- d) $-\frac{\pi i}{2}$

17 The Singular Point of the function $f(z) = \frac{z+4}{z^2-9z+20}$ are.

- a) $z = 4, 5$
- b) $z = -4, 5$
- c) $z = 4, -5$
- d) $z = -4, -5$

18 A point at which a function $f(z)$ ceases to be analytic is called.

- a) zero
- b) singularity
- c) pole
- d) limit point

19 $f(z) = \bar{z}$ is

- a) Continuous for every z , not differentiable for any z .
- b) Continuous for some values of z , differentiable for every z .
- c) discontinuous for every z , differentiable for every z .
- d) neither continuous nor differentiable.

20 If a function $f(z)$ is analytic in the entire complex plane then $f(z)$ is called.....

- a) Analytic continuation
- b) Meromorphic function
- c) Integral function
- d) Harmonic function

21 An analytic function with constant modulus is

- a) variable

- b) constant
- c) is zero
- d) does not exist

22 A continuous arc without multiple points is called a

- a) Jordan arc
- b) continuous arc
- c) contour
- d) rectifiable arc

23 If an entire function has a pole of order five at infinity then the polynomial of degree

- a) 4
- b) 5
- c) 6
- d) 0

24 To evaluate the integrals of the type $\int_0^{2\pi} \phi(\cos \theta, \sin \theta)$, the contour used is.

- a) Any circle
- b) semi circle
- c) unit circle
- d) rectangle

25 If principal part of Laurent's series consists of no terms then singularity is called

- a) removal Singularity
- b) pole
- b) essential Singularity
- d) irremovable Singularity

26 The value of $\int_c \frac{dz}{z}$, where c is the circle with centre at the origin and radius r is

- a) $\log r$
- b) πi
- c) $2\pi i$
- d) $\frac{\pi i}{2}$

- 27 Every analytic function in a simply connected domain
- a) Possesses a definite integral
 - b) Possesses a indefinite integral
 - c) Does not Possesses an indefinite integral
 - d) None of these
- 28 If $f(z) = u + iv$ to be analytic then Cauchy-Riemann equations are.....
- a) $u_x = v_y, u_y = -v_x$
 - b) $u_x = -v_y, u_y = v_x$
 - c) $u_x = v_y, u_y = v_x$
 - d) $u_x = -v_y, u_y = -v_x$
- 29 If c is the circle $|z - a| = r$, then $\int_c \frac{1}{(z-a)^n} dz = 2\pi i$, when
- a) $n \neq 1$
 - b) $n = 0$
 - c) $n = 1$
 - d) none of these
- 30 If $f(z) = u + iv$ is analytic function then
- a) both u & v are harmonic functions
 - b) both u & v need not to be harmonic function
 - c) u is harmonic function but v is not harmonic function
 - d) v is harmonic function but u is not harmonic function
- 31 If in the principal part of Laurent's series consist of an infinite number of terms then singularity is called.....
- a) removable singularity
 - b) essential singularity
 - c) pole
 - d) non-essential singularity
- 32 If $f(z)$ has a pole at $z = a$, then as $z \rightarrow a$,
- a) $|f(z)| \rightarrow 0$
 - b) $|f(z)| \rightarrow \infty$

- c) $|f(z)| \rightarrow \infty$
- d) $|f(z)| \rightarrow -\infty$

33 A function whose only singularities in the entire complex plane are poles, is called

- a) analytic function
- b) homomorphic function
- c) memomorphic function
- d) regular function

34 Function e^z has at $z = \infty$ is

- a) isolated singularity
- b) a pole
- c) an infinite point
- d) an isolated essential singularity

35 What kind of a) simple pole

function $\frac{1}{\sin z - \cos z}$ at $z = \frac{\pi}{4}$ is

- b) double pole
- c) singularity
- d) none of these

36 Residue of $f(z)$ at a simple pole $z = a$ is

- a) $\lim_{z \rightarrow a} z f(z)$
- c) $\lim_{z \rightarrow a} (z - a) f(z)$
- b) $\lim_{z \rightarrow a} \frac{f(z)}{z - a}$
- d) $\lim_{z \rightarrow a} \frac{z - a}{f'(z)}$

37 Residue of $\frac{z^2 - 2z}{(z+1)^2(z^2+4)}$ at double pole as $z = -1$ is

- a) $4/5$
- b) $-4/5$
- c) $-14/25$
- d) $14/25$

38 To integrate $\int_{-\infty}^{\infty} f(x) dx$ we should use

- a) circular contour

- b) indented semi-circular
- c) rectangular contour
- d) unit circle

39 Residue of $f(z)$ at $z = \infty$ is

- a) $\frac{1}{2\pi i} \int_c f(z) dz$
- b) $-\frac{1}{2\pi i} \int_c f(z) dz$
- c) $2\pi i \int_c f(z) dz$
- d) $-2\pi i \int_c f(z) dz$

40 The value of $\int_c \frac{e^z}{z-2}$ at $|z| = 1$ is

- a) $2\pi i$
- b) 0
- c) $2\pi i z_0$
- d) $4\pi i$

41 The value of the integral $\int_c \frac{dz}{z-2}$ where c is $|z| = 3$

- a) $2\pi i$
- b) $-2\pi i$
- c) $-\pi i$
- d) $-\frac{\pi i}{2}$

42 Function $f(z) = xy + iy$ is

- a) Everywhere continuous and analytic
- b) Everywhere continuous but not analytic
- c) Discontinuous but analytic everywhere
- d) neither continuous nor analytic.

43 A continuous arc without multiple points is called a

- a) Jordan arc
- b) continuous arc
- c) contour
- d) rectifiable arc

- 44 Consider two statements
- (i) Functions satisfying Laplace's equation is a harmonic
 - (ii) An analytic function with constant modulus in a domain is constant
- a) Only (i) is true
 - b) Only (ii) is true
 - c) Both (i) and (ii) are true
 - d) Both (i) and (ii) are false
- 45 To integrate $\int_0^{\infty} \frac{dx}{1+x^2}$ we will use a contour.
- a) real axis & unit circle $|z| = 1$
 - b) real axis & lower half of circle $|z| = R$
 - c) real axis & upper half of circle $|z| = R$
 - d) circle $|z| = R$
- 46 The limit point of the poles of a function $f(z)$ is
- a) a pole
 - b) an isolated singularity
 - c) a non isolated singularity
 - d) a non isolated essential singularity
- 47 If $f(z)$ is an analytic function of z & if $f'(z)$ is continuous at each point within & on a closed contour c then
- a) $\int_c f(z) dz = 2\pi i$
 - b) $\int_c f(z) dz = 0$
 - c) $\int_c f(z) dz = \pi i$
 - d) $\int_c f(z) dz \neq 0$
- 48 If an entire function has a pole of order 4 at infinity then it is a polynomial of degree
- a) 5
 - b) 4
 - c) Zero
 - d) infinity
- 49 The value of $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)$ is
- a) $2\left(\frac{\partial^2}{\partial z \partial \bar{z}}\right)$
 - b) $4\left(\frac{\partial^2}{\partial z \partial \bar{z}}\right)$

c) $-2 \left(\frac{\partial^2}{\partial z \partial \bar{z}} \right)$

d) $-4 \left(\frac{\partial^2}{\partial z \partial \bar{z}} \right)$

50 The analytic functions are called

- a) isomorphic
- b) homomorphic
- c) holomorphic
- d) conformal

Questions for 8 Marks

- 1) State and prove necessary conditions for $f(z)$ to be analytic.
- 2) If $f(z) = u(x, y) + iv(x, y)$ is analytic function and $z = re^{i\theta}$ is a polar form of z then show that Cauchy- Riemann equation in polar form are $\frac{\partial u}{\partial r} = \frac{1}{r} \frac{\partial v}{\partial \theta}$ and $\frac{\partial v}{\partial r} = -\frac{1}{r} \frac{\partial u}{\partial \theta}$
- 3) If $f(z)$ is analytic in a simply connected domain D except at finite number of poles $z_1, z_2, \dots, z_n$ within the closed contour and continuous on boundary which is a rectifiable Jordan curve then prove that $\int_C f(z) dz = 2\pi i \sum_{k=1}^n \text{Res}(z = z_k)$ hence evaluate $\int_C \frac{dz}{z^3(z+3)}$ over the circle $C: |z| = 1$.
- 4) Explain exact differential equation method for construction of an analytic function. Hence construct analytic function for $u = e^x \cos y$
- 5) If $f(z)$ and $g(z)$ are analytic inside and on a simple closed curve C , if $|g(z)| < |f(z)|$ on C then show that $f(z)$ and $f(z) + g(z)$ both have same number of zeros inside C .
- 6) Explain Milne-Thomson Method for construction of an analytic function by considering both cases.
- 7) If $f(z) = \frac{x^3(1+i) - y^3(1-i)}{x^2 + y^2}$, $z \neq 0$ and $f(z) = 0$, $z = 0$ then show that $f(z)$ is continuous and satisfy Cauchy Riemann equations at origin and $f'(0)$ does not exist.

- 8) If $f(z)$ is analytic in a simply connected domain D except at finite number of poles $z_1, z_2, \dots, z_n$ within the closed contour and continuous on boundary which is a rectifiable Jordan curve then prove that $\int_C f(z)dz = 2\pi i \sum_{k=1}^n \text{Res}(z = z_k)$ hence evaluate $\int_C \frac{e^{az}}{z+1} dz$ over the circle $C: |z| = 2$.
- 9) State and prove Cauchy theorem for simply connected domain.
- 10) State and prove sufficient conditions for $f(z)$ to be analytic.
- 11) If a domain D is bounded by system of closed rectifiable curves $C_1, C_2, \dots, C_k$ and $f(z)$ is analytic in domain D and continuous on $C_1, C_2, \dots, C_k$ then show that $\int_{C_1} f(z)dz + \int_{C_2} f(z)dz + \dots + \int_{C_k} f(z)dz = 0$
- 12) If $f(z)$ is analytic in domain D then show that $f(z)$ has derivatives of all orders at any point $z = a$ and all of which are analytic in domain D , their values is given by $f^n(a) = \frac{n!}{2\pi i} \int_C \frac{f(z)}{(z-a)^{n+1}} dz$ where C is any closed curve surrounding the point $z = a$.
- 13) State and prove Cauchy integral formula for simply connected domain.
- 14) Explain the method to evaluate the integral of the type $\int_{-\infty}^{\infty} f(z)dz$ hence show that $\int_0^{\infty} \frac{dz}{1+z^2} = \frac{\pi}{2}$.
- 15) State and prove Cauchy Residue theorem.

Questions for 4 Marks

- 1) Show that $f(z) = |z|^2$ is continuous everywhere but nowhere differentiable except at the origin.
- 2) If $f(z) = u + iv$ is an analytic function. If $f'(z) = 0$ then show that $f(z)$ is constant.
- 3) If $f(z) = u + iv$ is an analytic function then show that $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = 4 \frac{\partial^2}{\partial z \partial \bar{z}}$.

- 4) If $u = e^x \cos y$ is harmonic function then find the harmonic conjugate of u by using exact differential equation method and hence construct the analytic function.
- 5) Evaluate $\int \bar{z} dz$ along the line from $z = 0$ to $z = 2i$ and then from $z = 2i$ to $z = 4 + 2i$.
- 6) Find the value of integral $\int_0^{1+i} (x - y + ix^2) dz$ along the straight-line from $z = 0$ to $z = 1 + i$.
- 7) Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in a Laurent's series for the region $|z| > 3$.
- 8) Show that $\int_C \bar{z} dz$ along a semi-circular path from $z = -a$ to $z = a$ lies above the x axis is $-\pi ia^2$.
- 9) If $f(z) = \frac{z-4}{(z-3)(z-5)^2}$ then find the residues at corresponding poles.
- 10) Prove that analytic function with constant modulus is constant.
- 11) Prove that all the roots of $z^7 - 5z^3 + 12 = 0$ lie between the circle $|z| = 1$ and $|z| = 2$.
- 12) Show that the real and imaginary part of $f(z) = e^z$ are harmonic.
- 13) Evaluate $\int_0^{2\pi} \frac{1+2 \cos \theta}{5+4 \cos \theta} d\theta$.
- 14) If $f(z) = u + iv$ is an analytic function then show that $u(x, y) = c_1$ and $v(x, y) = c_2$ represent orthogonal family of curves.
- 15) Evaluate $\int_0^{1+i} z^2 dz$.
- 16) If $f(z) = u + iv$ is an analytic function then find $f(z)$ in terms of z where $u - v = (x - y)(x^2 + 4xy + y^2)$.
- 17) Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in a Laurent's series for the region $1 < |z| < 3$.
- 18) If $f(z) = \frac{z^4}{z^2 + a^2}$ then find the residues at corresponding poles.

19) Show that $\int_C \bar{z} dz$ along a semi-circular path from $z = -a$ to $z = a$ lies below the x axis is $\pi i a^2$.

20) Find the type of singularities of $f(z) = \frac{\cot \pi z}{(z-a)^2}$ at $z = a$ and $z = \infty$.

21) If $u = \log(x^2 + y^2)$ then construct the analytic function.

22) Evaluate $\int_C \frac{z^4}{z-3i} dz$ by using Cauchy integral formula along the curve $|z-2| < 5$.

23) Prove that all the roots of $z^7 - 5z^3 + 12 = 0$ lie between the circle $|z| = 1$ and $|z| = 2$.

24) What kind of singularities exist for the function $f(z) = \frac{1-e^z}{1+e^z}$ at $z = \infty$.

25) Find the value of integral $\int_0^{1+i} (x - y + ix^2) dz$ along the straight-line from $z = 0$ to $z = 1 + i$.

26) Find the residue of $f(z) = \frac{z^2}{(z^2+1)^2}$ at $z = \infty$.

27) Expand $f(z) = \frac{1}{z(z^2-3z+2)}$ in a Laurent's series for the region $0 < |z| < 1$.

28) If $f(z) = \frac{z-4}{(z-3)(z-5)^2}$ then find the residues at corresponding poles.

29) Evaluate $\int_C \frac{e^{az}}{z+1} dz$ over the circle $C: |z| = 2$.

30) Find the type of singularities of $f(z) = \tan\left(\frac{1}{z}\right)$ at $z = 0$.

31) Write Cauchy- Riemann equations in cartesian form.

32) Define singular point

33) How to calculate the singularity of function $f(z)$ at infinity?

34) Define pole.

35) What is the isolated singularity?

36) Define the Jordan curve.

37) Define complex valued function.

- 38) Define meromorphic function
- 39) Define limit of a complex valued function
- 40) Write Cauchy- Riemann equations in polar form.
- 41) Define principal argument of complex number
- 42) Define harmonic function
- 43) What is the modulus and argument of $z = -i$.
- 44) Define removable singularity.
- 45) Define regular point.
- 46) Define simple curve.
- 47) Write the solution of exact differential equation $Mdx + Ndy = 0$.
- 48) Define entire function.
- 49) Write the equation of circle whose center is at c and radius a in complex plane.
- 50) Define analytic function.
- 51) Write statement of Greens theorem.
- 52) Define essential singularity.
- 53) Define cross cut.
- 54) Define smooth curve.
- 55) Define residue of an isolated singularity.

