Objective Questions

Sequence of Real Numbers

1. The sequence $\left\{\log\left(\frac{1}{n}\right)\right\}_{n=1}^{\infty}$ ……

   a) converges to 1 c) diverges to ∞
   b) converges to e d) diverges to $-\infty$

2. If $C = \{C_n\} = \{\sqrt{n}\}$ and $N = \{n_i\} = \{i^4\}, i \in N,$ then $C \circ N = ...$

   a) $\{i\}$ b) $\{i^2\}$ c) $\{i^3\}$ d) $\{i^4\}$

3. If $0 < x < 1,$ then the sequence $\{x^n\}_{n=1}^{\infty}$.....

   a) converges to 0 c) diverges to ∞
   b) converges to 1 d) diverges to $-\infty$

4. If $S = \{S_n\} = \left\{\frac{1}{n}\right\}$ and $N = \{n_i\} = \{2^i\}$, then $S \circ N = ...$

   a) 2^i b) 2^{-i} c) 2 d) 2^{-1}

5. A sequence $\{S_n\}_{n=1}^{\infty}$ is called null sequence if it ….

   a) converges to 0 c) diverges to ∞
   b) converges to 1 d) diverges to $-\infty$

6. The sequence $\left\{\frac{n}{n+1}\right\}_{n=1}^{\infty}$ is …

   a) non-decreasing c) bounded
   b) monotonic d) all the above

7. The sequence $\{(-1)^n\}_{n=1}^{\infty}$ is …

   a) not bounded above c) not bounded below
   c) monotonic d) not monotonic

8. $\lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)^{2n} = ...$

   a) e b) $\frac{1}{e}$ c) $\sqrt{e}$ d) e^2

9. $\lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)^{n/2} = ...$

   a) e b) $\frac{1}{e}$ c) $\sqrt{e}$ d) e^2

10. $\lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)^{n-1} = ...$

a) e b) e^2 c) e^3 d) $\sqrt{e}$

11. $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n\pm 50} = ...$

a) e b) e^2 c) e^3 d) $\sqrt{e}$

12. Limit of convergent sequence is l are ….

a) unique
b) more than one
c) two
d) none of these

13. Every convergent sequence is ….

a) bounded above
b) bounded below
c) bounded
d) none of these

14. A non-decreasing sequence which is bounded above is ….

a) divergent
b) oscillatory
c) convergent
d) none of these

15. A non-increasing sequence which is bounded below is ….

a) divergent
b) oscillatory
c) convergent
d) none of these

16. A non-decreasing sequence which is not bounded above …

a) converges to 0
b) converges to 1
c) diverges to ∞
d) diverges to $-\infty$

17. A non-increasing sequence which is not bounded below …

a) converges to 0
b) converges to 1
c) diverges to ∞
d) diverges to $-\infty$

18. If $\{S_n\}_{n=1}^{\infty}$ is a sequence of non-negative real numbers and $\lim_{n\to\infty} S_n = L$, then …

a) $L=0$ b) $L=1$ c) $L\le 0$ d) $L\ge 0$

19. If $\{S_n\} = \{(-1)^n\}_{n=1}^{\infty}$, then $\lim_{n\to\infty} Sup\ S_n = ...$

a) 0 b) 1 c) -1 d) 2

20. If $\{S_n\} = \{(-1)^n\}_{n=1}^{\infty}$, then $\lim_{n\to\infty} Inf\ S_n = ...$

a) 0 b) 1 c) -1 d) 2

21. If $\{S_n\} = \{1,2,3,4,1,2,3,4,....\}$, then $\lim_{n\to\infty} Sup\ S_n = ...$ and $\lim_{n\to\infty} Inf\ S_n = ...$

a) 4, 1 b) 1, 4 c) 4, 4 d) 1, 1

22. If $\{S_n\} = \left\{\sin\frac{n\pi}{2}\right\}_{n=1}^{\infty}$, then $\lim_{n\to\infty} Sup\ S_n = ...$ and $\lim_{n\to\infty} Inf\ S_n = ...$

a) 1, 1 b) 1, -1 c) -1, 1 d) -1, -1

23. If $\{S_n\} = \{1,-1,1,-2,1,-3,....\}$, then $\lim_{n\to\infty} Sup\ S_n = ...$

a) 0 b) 1 c) -1 d) 2

24. If $\{S_n\}_{n=1}^{\infty}$ is not bounded above, then $\lim_{n\to\infty} Sup\ S_n = ...$

a) 1 b) -1 c) ∞ d) $-\infty$

25. If $\{S_n\} = \{10^{100}, 1, -1, 1, -1, 1, -1, ...\}$, then $\lim_{n\to\infty} Sup\ S_n = ...$

a) 1 b) -1 c) 10 d) 10^{100}

26. If $\{S_n\} = \{n\}_{n=1}^{\infty}$, then $\lim_{n\to\infty} Inf\ S_n = ...$

a) 1 b) 2 c) 3 d) ∞

27. The sequence $\{S_n\} = \{1, 0, 1, 0, 1, 0, ...\}$ is (C, 1) summable to ….

a) 1 b) ½ c) 2 d) 0

28. A sequence $\{S_n\} = \{(-1)^n\}$ is (C, 1) summable to ….

a) 0 b) 1 c) 2 d) -2

29. A sequence $\{S_n\} = \{n\}$ is ….

a) (C, 1) summable to 0
b) (C, 1) summable to 1
c) (C, 1) summable to-1
d) not (C, 1) summable

30. A sequence $\{S_n\}_{n=1}^{\infty} = \{1\}$ is (C, 1) summable to ….

a) 0 b) 1 c) -1 d) 2

31. The sequence $\{a_n\}_{n=1}^{\infty}$ where, $a_n = \dfrac{n^2+1}{2n^2+5}$, converges to ….

a) 1/5 b) ½ c) 3/2 d) 1

32. The sequence $\{4^n\}_{n=1}^{\infty}$ ….

a) converges to 4
b) converges to -4
c) diverges to ∞
d) diverges to $-\infty$

33. The sequence $\{n^2\}_{n=1}^{\infty}$ is ….

a) bounded above
b) bounded below
c) bounded
d) oscillatory

34. The sequence $\left\{\left(-\dfrac{3}{4}\right)^n\right\}_{n=1}^{\infty}$ ….

a) is convergent
b) diverges to ∞
c) diverges to $-\infty$
d) is oscillatory

35. A lower bound of the sequence $\{2+n^2\}_{n=1}^{\infty}$ is …

a) 1 b) 4 c) e d) do not exists

36. The sequence $\{1, 1, 1, 1, ...\}$ is ….

a) (C, 1) summable
b) divergent
c) not (C, 1) summable
d) none of these

37. The sequence $\left\{(-1)^n n\right\}_{n=1}^{\infty}$ is ….

a) bounded above
b) bounded below
c) bounded above as well as bounded below
d) neither bounded above nor bounded below

38. The decreasing sequence is ….

a) $\left\{2+\frac{1}{n}\right\}_{n=1}^{\infty}$
b) $\left\{2-\frac{1}{n}\right\}_{n=1}^{\infty}$
c) $\left\{1-\frac{1}{n}\right\}_{n=1}^{\infty}$
d) $\left\{-\frac{1}{n}\right\}_{n=1}^{\infty}$

39. For the given sequence $\left\{(-1)^n\left(1+\frac{1}{n}\right)\right\}_{n=1}^{\infty}$ which one of following statement is correct?

a) limit superior = limit inferior
b) neither limit superior nor limit inferior exists.
c) limit superior =1 and limit inferior = -1
d) limit superior =1 and limit inferior =0

40. The supremum of the set $\left\{1,\frac{1}{2},\frac{1}{3},\frac{1}{4},....\right\}$ is ….

a) 0
b) 1
c) ½
d) ¼

41. In a sequence $\{s_n\}$, $s_{n+1} \geq s_n$ for all $n \in I$, then sequence is ….

a) monotonically increasing
b) monotonically decreasing
c) strictly monotonically increasing
d) none of these

42. If the sequence $\{s_n\}_{n=1}^{\infty}$ converges to 0, then the sequence $\left\{(-1)^n s_n\right\}_{n=1}^{\infty}$ converges to …

a) $-\infty$
b) g. l. b. $\{s_n, s_{n+1}, s_{n+2},.....\}$
c) l. u. b. $\{s_n, s_{n+1}, s_{n+2},.....\}$
d) ∞

43. If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are bounded sequences of real numbers then ….

a) $\lim_{n\to\infty} \sup[s_n + t_n] \neq \lim_{n\to\infty} \sup s_n + \lim_{n\to\infty} \sup t_n$
b) $\lim_{n\to\infty} \sup[s_n + t_n] \geq \lim_{n\to\infty} \sup s_n + \lim_{n\to\infty} \sup t_n$
c) $\lim_{n\to\infty} \sup[s_n + t_n] \leq \lim_{n\to\infty} \sup s_n + \lim_{n\to\infty} \sup t_n$
d) $\lim_{n\to\infty} \sup[s_n + t_n] \geq \lim_{n\to\infty} \inf[s_n + t_n]$

44. The sequence $\{x_n\}_{n=1}^{\infty}$ converges to 0 if ….

a) $1 \leq x < \infty$
b) $1 < x < \infty$
c) $0 < x \leq 1$
d) $0 < x < 1$

45. Given the sequence $\left\{\frac{n}{n+1}\right\}_{n=1}^{\infty}$ and an arbitrary small positive number ε, then the value of a positive integer m such that $\left|\frac{n}{n+1}-1\right|<\varepsilon$ whenever $n \geq m$ must satisfies the relation ….

a) $m \leq \left(\frac{1}{\varepsilon}\right)-1$

b) $m < \left(\frac{1}{\varepsilon}\right)-1$

c) $m \geq \left(\frac{1}{\varepsilon}\right)-1$

d) $m > \left(\frac{1}{\varepsilon}\right)-1$

Infinite Series

46. A necessary condition for the convergence of the infinite series $\sum u_n$ is ….

a) $\lim_{n\to\infty} u_n \neq 0$

b) $\lim_{n\to\infty} u_n = 0$

c) $\lim_{n\to 0} u_n = 0$

d) $\lim_{n\to 0} u_n \neq 0$

47. The series $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+....$ is ….

a) convergent

b) divergent

c) neither convergent nor divergent

d) none of these

48. The positive p-series $\sum \frac{1}{n^p}$ is divergent for …..

a) $p<1$ b) $p \geq 1$ c) $p \leq 1$ d) $p>1$

49. A positive term series converges iff the sequence of partial sums is ….

a) bounded below

b) bounded above

c) both a) and b)

d) none of these

50. Consider the statements (i) $\sum \frac{1}{n}$ is divergent (ii) $\sum \frac{1}{n^2}$ is convergent then ….

a) both (i) and (ii) are true

b) both (i) and (ii) are false

c) only (i) is true

d) only (ii) is true

51. The series $\sum \left(\frac{3}{2}\right)^n$ is ….

a) convergent

b) divergent

c) neither convergent nor divergent

d) none of these

52. If $\sum u_n$ is series of positive terms with $\lim_{n\to\infty} \frac{u_{n+1}}{u_n} = L$, then the series diverges if ….

a) L<1 b) L>1 c) L=0 d) none of these

53. The series $\sum\left(1+\frac{1}{n}\right)^{n^2}$ is … series.

a) convergent
b) divergent
c) neither convergent nor divergent
d) cannot be decided

54. The series $\sum\frac{1}{n(\log n)^p}$ converges for ….

a) $p<1$
b) $p\le 1$
c) $p>1$
d) $p=1$

55. The series $\sum\sin\frac{1}{n^2}$ is ….

a) convergent
b) divergent
c) cannot be decided
d neither a) nor b)

56. The series $\sum\left(1+\frac{1}{\sqrt{n}}\right)^{-n^{3/2}}$ converges to ….

a) e
b) 0
c) $\frac{1}{e}$
d) 1

57. The series $\sum\frac{x^n}{n(n+1)}, x>0$ converges for ….

a) $x<1$
b) $x>1$
c) $x\ge 1$
d) $x\le 1$

58. The series $1+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+...$ is ….

a) divergent
b) convergent
c) neither a) nor b)
d) cannot be decided

59. The series $\sum\frac{n!}{n^n}$ converges to …

a) e
b) 1
c) $\frac{1}{e}$
d) 0

60. The series $\sum\frac{1}{\sqrt{n}}$ is ….

a) converges to 0
b) converges to 1
c) divergent
d) none of these

61. For the convergence of series $\sum u_n$, the condition $\lim_{n\to\infty} u_n=0$ is …

a) only necessary
b) sufficient
c) necessary and sufficient
d) none of these

62. The series $\sum\cos\left(\frac{\pi}{n}\right)$ is ….

a) convergent
b) divergent
c) neither a) nor b)
d) cannot be decided

63. The series $\sum \frac{1}{\sqrt{n+1}-\sqrt{n}}$ is ….

a) converges to 0
b) converges to 1
c) divergent
d) none of these

64. The series $\sum \frac{1}{2n^2+3n+5}$ is ….

a) convergent
b) divergent
c) neither a) nor b)
d) cannot be decided

65. The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if ….

a) $p<1$ b) $p \geq 1$ c) $p \leq 1$ d) $p>1$

66. The geometric series $\sum_{n=1}^{\infty} x^{n-1}$ is convergent if ….

a) $x>1$ b) $0 \leq x<1$ c) $x \neq 0$ d) $x=1$

67. The series $\sum_{n=1}^{\infty} \frac{1}{n}$ is ….

a) convergent b) divergent c) oscillatory d) none of these

68. The series $\sum_{n=1}^{\infty} \frac{1}{n!}$ is ….

a) convergent b) divergent c) oscillatory d) none of these

69. If $\sum_{n=1}^{\infty} a_n$ is a series of positive terms and $\lim_{n \to \infty} \sqrt[n]{a_n} = l$, then the series is convergent if ….

a) $l>0$ b) $l=1$ c) $l>1$ d) $l<1$

70. A series $\sum_{n=1}^{\infty} a_n$ is convergent if and only if …. is convergent.

a) $\left\{\sum_{k=1}^{\infty} a_k\right\}_{n=1}^{\infty}$ b) $\left\{\sum_{k=1}^{\infty} {a_k}^2\right\}_{n=1}^{\infty}$ c) $\{a_k\}_{k=1}^{\infty}$ d) $\{|a_n|\}_{n=1}^{\infty}$

71. Let $\sum a_n$ be a series of non-negative terms and $s_n = a_1 + a_2 + ... + a_n$. Then the series is convergent if and only if …

a) $\{s_n\}$ is not bounded above
b) $\{a_n\}$ is bounded above
c) $\{a_n\}$ is convergent
d) $\{s_n\}$ is bounded above

72. Which of the following series converges?

a) $\sum_{n=1}^{\infty} 1$ b) $\sum_{n=1}^{\infty} \frac{1}{n}$ c) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n}$ d) $\sum_{n=1}^{\infty} (-1)^{n+1}$

73. If $\sum u_n$ and $\sum v_n$ are two positive term series that $\lim_{n\to\infty} \frac{u_n}{v_n} = l$ is non-zero finite number, then …..

a) if $\sum u_n$ converges then $\sum v_n$ converges

b) if $\sum v_n$ converges then $\sum u_n$ converges

c) if $\sum u_n$ diverges then $\sum v_n$ converges

d) both series converges or diverges together

74. The sum of the series $\sum_{n=0}^{\infty} \frac{1}{n!}$ is ….

a) e b) 0 c) 1 d) -1

75. The sum of the series $\sum_{n=0}^{\infty} \frac{1}{e^n}$ is ….

a) $\frac{1}{e-1}$ b) $\frac{1}{e}$ c) $\frac{e}{e-1}$ d) e

76. The series 0.6+0.06+0.006+…. is ….

a) convergent to 0.6 c) convergent to 1/3

b) convergent to 2/3 d) divergent

77. The limit of the sequence defined by $a_n = \ln(2n^3 + 2) - \ln(5n^3 + 2n^2 + 4)$ is …

a) 0 **b)** $\ln\left(\frac{2}{5}\right)$ c) $-\ln\left(\frac{2}{5}\right)$ d) $\frac{2}{5}$

78. All possible values of x for which the series $\sum_{n=0}^{\infty} \frac{9 + x^n}{5^n}$ converges …

a) $|x| < 1$ b) $-3 < x < 3$ **c)** $-5 < x < 5$ d) $-3 < x < 5$

79. The limit of the sequence defined by $a_n = n^2 \cos\left(\frac{2}{n^2} + \frac{\pi}{2}\right)$ is …

a) **-2** b) -1 c) 0 d) 1

80. Which of the following series converge

$a_n = \frac{(2n+1)!}{(n+4)!}$, $b_n = \frac{\pi^n}{n^{100}}$, $c_n = \frac{\ln(n^{10})}{\sqrt{n}}$, $d_n = \frac{n^4}{(n+1)!}$

a) $\{d_n\}$ only b) $\{a_n\},\{b_n\}$ only **c)** $\{c_n\},\{d_n\}$ **only** d) $\{a_n\},\{d_n\}$ only

81. The terms of the sequence $\{a_n\}$ are given by the formula

$$a_n = \frac{1}{3n^3} + \frac{2^2}{3n^3} + \frac{3^2}{3n^3} + ... + \frac{n^2}{3n^3}$$ the limit of the sequence is …

a) ½ b) 2/3 c) 1/6 **d) 1/9**

82. Determine the value of the series $\sum_{n=2}^{\infty} \frac{6}{n(n+3)}$ or conclude that it diverges to …

a) 0 b) 13/2 c) 5/3 **d) 13/6**

83. Determine the value of the series $\sum_{n=0}^{\infty} \frac{2^{n-2}+3^{n+1}}{4^n}$ or conclude that it diverges to …

a) **25/2** b) 9/2 c) 13/2 d) 37/4

84. Assume $\sum_{n=1}^{\infty} a_n$ is an infinite series with partial sum given by $S_N = 4 + \frac{2}{N}$. What is a_5 …

a) 3/10 b) -3/10 c) 3/10 **d) -1/10**

85. Which of the following series converge….

I. $\sum_{n=1}^{\infty} \frac{(-1)^n}{1+\frac{1}{n}}$, II. $\sum_{n=2}^{\infty} \frac{1}{\ln(n^4)}$, III. $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2+1}}$

a) I only b) II only c) III only **d) none of them**

86. The series $\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$. Find the value of the series $\sum_{n=2}^{\infty} \left(\frac{2}{n}\right)^4$ …

a) $\frac{8\pi^4}{45}$ b) $\frac{8\pi^4}{45} - 1$ **c)** $16\left(\frac{\pi^4}{90} - 1\right)$ d) $\frac{8\pi^4}{90}$

87. Which of the following series converge …

I. $\sum_{n=1}^{\infty} \frac{2^n+n^4}{4^n+n^2}$, II. $\sum_{n=1}^{\infty} \frac{4^n}{5^n+n}$ III. $\sum_{n=1}^{\infty} \frac{n!}{3^n}$

a) **I and II** b) I and III c) II and III d) All of them

88. Which of the following series converge….

I. $\sum_{n=1}^{\infty} \frac{\ln n}{e^n+2}$ II. $\sum_{n=1}^{\infty} \frac{\sin^2(n)}{n+n^{3/2}}$ III. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{2/3}}$

a) I and II b) I and III c) II and III **d) All of them**

89. Which of the following alternating series converge conditionally, but not absolutely …

I. $\sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt{n}\ln n}$ II. $\sum_{n=2}^{\infty} \frac{(-1)^n}{n(\ln n)^2}$ III. $\sum_{n=1}^{\infty} \frac{\cos(\pi n)}{2^{n-3}}$

a) None of them **b) I only** c) II only d) III only

90. For which value of p does the series $\sum_{n=1}^{\infty} \frac{e^n}{(2+e^{2n})^p}$ converge …

a) All values of p b) $-1 < p < 1$ c) $p \geq 1$ **d)** $p > \frac{1}{2}$

91. Let $\sum_{n=1}^{\infty} a_n$ be a series with partial sums S_N. Which of the following statements are always true …

I) If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges.

II) If $\sum_{n=1}^{\infty} a_n = L$, then $\lim_{n\to\infty} a_n = L$.

III) If $\sum_{n=1}^{\infty} a_n$ converges then, $\lim_{n\to\infty} a_n = 0$.

IV) If $\lim_{n\to\infty} S_N = L$, then $\sum_{n=1}^{\infty} a_n = L$.

a) I and II b) I and III **c) III and IV** d) II and III

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