Multiple Choice Questions

Functions and Countable Sets

1. If $A = \{x \in N \mid 2x - 1 \text{ is even}\}$ and $B = \{x \in R \mid x^2 - 9 = 0\}$ then which of the following statements is true.
   a) A is empty and B is empty
   b) A is nonempty and B is empty
   c) A is nonempty and B is nonempty
   d) A is empty and B is nonempty
2. If $A = \phi$ and $B = \{\phi\}$ then which of the following statements is true.
   a) $\phi \in A$ b) $A \subset B$ c) $B \subset A$ d) $B = \phi$
3. If $f : R \to R$ is given by $f(x) = x^2 + 1$, then $f^{-1}(-5)$ is
   a) -5 b) 5 c) ϕ d) 0
4. If $f : R \to R$ is given by $f(x) = x^2$ then the range of f is
   a) R c) N
   b) R_1 (the set of nonnegative real numbers) d) Z
5. If $f : R^+ \to R$ (where R^+ is the set of positive real numbers) is given by $f(x) = \log x$ then the set $\{x \mid f(x) = -2\}$ is
   a) ϕ b) -2 c) e^2 d) e^{-2}
6. If $f : R \to R$ is given by $f(x) = |x|$ then f is
   a) Injection b) surjection c) bijection d) none of the above
7. Which of the following statements is correct?
   a) $a \in \{a, b, c\}$ b) $\{a\} \in \{a, b, c\}$ c) $a \subset \{a, b, c\}$ d) $\{\{a\}, b, c\} \subseteq \{a, b, c\}$
8. If C and R be the sets of complex and real numbers respectively. If $f : C \to R$ is given by $f(z) = |z|$ then f is
   a) One-one b) onto c) one-one and onto d) neither one-one nor onto
9. The set $\{x \mid x < 7\}$ is the interval
   a) (0, 7) b) $(-\infty, 7)$ c) [0, 7] d) $(-\infty, 7]$
10. If $A = \{1, 2, \{3\}, (4, 5)\}$ then the number of elements in it is
    a) 3 b) 4 c) 5 d) 2
11. If P is the set of prime integers, then which of the following is true

a) $7 \in P$ b) $11 \notin P$ c) $9 \in P$ d) $\{7\} \subset P$.

12. Set of real numbers is …

A) Uncountable C) finite

B) Countable D) none of these

13. If f is a function from A into B with range of $f = B$ then f is called …

A) Onto C) one-one and onto

B) One-one D) none of these

14. A function $f : A \to B$ is called a one-one correspondence between A and B, if …

A) f is neither one-one nor onto

B) f is one-one and onto

C) f is one-one but not onto

D) f is not one-one but onto

15. If $f : A \to B$ and if $f(a_1) = f(a_2)$ implies $a_1 = a_2$ for all $a_1, a_2 \in A$, then f is called … function.

A) Onto C) one-one and onto

B) One-to- one D) none of these

16. Set of all rational numbers in $(0,1)$ is …

A) Finite C) countable

B) Uncountable D) none of these

17. If f and g are two functions with respective domains X and Y then g is called extension of f onto Y if …

A) $X \supset Y$ and if $f[X] = g[X]$, for all $x \in X$

B) $X \subset Y$ and if $f[X] = g[X]$, for all $x \in X$

C) $X \subset Y$ and if $f[X] \neq g[X]$, for all $x \in X$

D) $X \supset Y$ and if $f[X] \neq g[X]$, for all $x \in X$

18. A function $f : A \to B$ is called a one-one correspondence between A and B, if …

A) f is one-one but not onto

B) f is one-one and onto

C) f is not one-one but onto

D) f is neither one-one nor onto

19. The set of … is uncountable set.

A) Positive integers C) rational numbers

B) Integers D) real numbers

20. If $f(x)=1+\sin x\ (-\infty<x<\infty)$ and $g(x)=x^2\ (0\le x<\infty)$ then $g\circ f(x)=$ …

A) $1+\sin^2 x\quad(-\infty<x<\infty)$

B) $1+2\sin x+\sin^2 x\quad(0\le x<\infty)$

C) $1+\sin^2 x\quad(0\le x<\infty)$

D) $1+2\sin x+\sin^2 x\quad(-\infty<x<\infty)$

21. If $f:A\to B,\ X\subset A, Y\subset A$, then $f(X\cap Y)$ is …

A) Equal to $f(X)\cup f(Y)$

B) Equal to $f(X)\cap f(Y)$

C) Not necessarily equal to $f(X)\cap f(Y)$

D) Not necessarily equal to $f(X)\cup f(Y)$

22. The set of rational numbers is …

A) Countable C) finite

B) Uncountable D) none of these

23. If $g(x)=x^2(0\le x<\infty)$, then $g^{-1}(x)....(0\le x<\infty)$.

A) x^2 B) x C) $x^{3/2}$ D) $x^{1/2}$

24. If A is any non-empty subset of R that is bounded below, then A has … in R.

A) A greatest lower bound

B) A least upper bound

C) Upper bound

D) None of these

25. Let f be a real valued function described by $f(x)=x^2(-\infty<x<\infty)$. Then $f([0,3))=$ …

A) (0, 9) B) (0, 9] C) [0, 9) D) [0, 9]

Mathematical Induction

1. The validity of statement $p(n)$ is proved by using mathematical induction for even

   a) Real number b) integer c) natural number d) rational number

2. Well ordering principle states that every nonempty subset of natural number N has a ….. element.
   a) One b) infinite c) no d) least
3. The condition p(1) is true is ……. Condition for proving validity of p(n) by mathematical induction.
   a) Necessary b) sufficient
   b) Necessary and sufficient d) neither necessary nor sufficient
4. If p(n) is true for $n = n_0$ and p(k) is true implies p(k+1) is true, then this type of mathematical induction is called ……. Version of mathematical induction.
   a) First b) second c) regular d) principal
5. By mathematical induction, the result $2^n < n!$ is true for all…..
   a) $n \geq 2$ b) $n \geq 3$ c) $n \geq 4$ d) $n \geq 1$
6. The result $n^2 < 2^n$ for all $n \in N$ is not true for all $n \in N$.
   a) Not true for n=1 c) both a) and b)
   b) Truth for n=k does not imply truth for n=k+1 d) none of these
7. By using second version of mathematical induction the result
   $$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + ... + \frac{7}{\sqrt{n}} > \sqrt{n}$$ is true for all ….
   a) $n \geq 1$ b) $n \geq 2$ c) $n \geq 3$ d) $n \geq 4$
8. The result for every subset S of N if $1 \in S$ and for every $K \in N$, $\{1,2,3,.....\} \leq S$ then S=N is called ……
   a) Principle of Mathematical induction
   b) First version of Mathematical induction
   c) second version of Mathematical induction
   d) principle of strong induction
9. The result for every nonempty set S of N if $1 \in S$ and for every $K \in N, K \in S$ then $K+1 \in S$ then S=N is called ……..
   a) Principle of Mathematical induction
   b) First version of Mathematical induction
   c) second version of Mathematical induction
   d) principle of strong induction
10. The result n=n+2 is false by mathematical induction because …….
   a) It is not true for n=1 c) both a) and b)
   b) Truth for n=k does not imply truth for n=k+1 d) none of these

Countable Sets

1. Every 1-1 correspondence is ……..
   a) One-one b) onto c) countable d) all a), b), c).
2. The function $f(x) = x^2, x \in R$ is ……..
   a) One-one b) onto
   c) neither one-one nor onto d) none of these
3. The set of rational numbers is ……..
   a) Finite b) countable c) uncountable d) none of these
4. The set of natural numbers is ……
   a) Finite b) countable c) uncountable d) none of these
5. The set of integers is ……
   a) Finite b) countable c) uncountable d) none of these
6. The set of real numbers is …….
   a) Finite b) countable c) uncountable d) none of these
7. The closed interval [0, 1] is ……
   a) Countable c) neither countable nor uncountable
   b) Uncountable d) finite
8. The open interval (0, 1) is ……
   a) Countable c) neither countable nor uncountable
   b) Uncountable d) finite
9. The set of rational numbers in [0, 1] is ……
   a) Countable c) neither countable nor uncountable
   b) Uncountable d) finite
10. The Cartesian product $Z \times Z$ where Z is the set of integers is ……..
    a) Countable c) neither countable nor uncountable
    b) Uncountable d) finite
11. The Cartesian product of two countable sets is …….
    a) Countable c) neither countable nor uncountable
    b) Uncountable d) finite
12. The function $f(x) = \cos x$ in [0, 1] is ……
    a) One-one b) onto c) one-one and onto d) none of these

13. The union of countable sets is ……..
    a) Countable
    b) Uncountable
    c) neither countable nor uncountable
    d) none of these
14. If the set A is equivalent with set of integers then the set A is ……..
    a) Finite b) countable c) uncountable d) none of these
15. The set of all ordered pairs of integers is ……
    a) Finite b) countable c) uncountable d) none of these
16. The set of all polynomial functions with integer coefficients is ……
    a) Finite b) countable c) uncountable d) none of these

The Real Numbers

1. Greatest lower bound of set of all positive even integers is …….
   a) 0 b) 2 c) 1 d) none of these
2. The least upper bound of the set $\left\{\frac{1}{n} \mid n \in N\right\}$ is …
   A) 1 B) 0 C) -1 D) none of these
3. Between any two distinct real numbers there exists ….
   A) Only one rational number
   B) Finite number of rational numbers
   C) Infinitely many rational numbers
   D) None of these
4. If u is an upper bound of a set A of real numbers and $u \in A$, then u is …
   A) Infimum of A
   B) Both infimum and supremum of A
   C) supremum of A
   D) neither infimum nor supremum
5. A subset A of real numbers is said to be bounded if it is …
   A) Bounded above
   B) Bounded above as well as bounded below
   C) bounded below
   D) none of these
6. Supremum of the set $\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots.\right\}$ is …
   A) 0 B) 1 C) ½ D) ¼
7. Every non-empty subset S of R which has a lower bound has …

A) Infimum B) supremum C) no infimum D) none of these

8. If x and y are real numbers, which of the following is always true ?

A) $|x-y| \leq |x|-|y|$ C) $|x-y| \geq |x|-|y|$

B) $|x-y| \geq |x|+|y|$ D) $|x-y| = |x|-|y|$

9. Any non-empty subset of real numbers which is bounded below has …

A) Infimum C) supremum

B) Both infimum and supremum of A D) neither infimum nor supremum

10. The supremum of a set A of real numbers, if it exists is …

A) Unique C) three

B) Two D) none of these

11. The least upper bound of the set $\left\{\frac{3n+2}{2n+1} \mid n \in N\right\}$ is …

A) 3/5 B) 5/3 C) 1/5 D) 1/3

12. The greatest lower bound of the set $\left\{x \in Q \mid x = \frac{(-1)^n}{n}, n \in N\right\}$ is …

A) 0 B) 1 C) -1 D) 2

13. For what value of m, such that $|f(x)| \leq m$ for all x satisfying $2 \leq x \leq 3$ where f is defined by

$$f(x) = \frac{2x^2+3x+1}{2x-1} \text{ for } 2 \leq x \leq 3.$$

A) 23/3 B) 25/3 C) 28/3 D) 29/3

14. If a and b are real numbers which one of the following is always true?

A) $|a+b| = |a|+|b|$ C) $|a+b| \leq |a|+|b|$

B) $|a+b| \geq |a|+|b|$ D) $|a+b| < |a|+|b|$

15. If a and b are real numbers then …

A) $|a-b| \geq |a|+|b|$ C) $|a-b| = |a|+|b|$

B) $|a-b| > |a|+|b|$ D) $|a-b| \leq |a|+|b|$

16. If … then $|a+b| = |a|+|b|$, where $a,b \in R$

A) $ab \geq 0$ B) $ab = 0$ C) $ab < 0$ D) $ab \leq 0$

17. If a and b are real numbers then …

A) $|a-b| \leq |a|-|b|$ C) $|a-b| \geq |a|-|b|$

B) $|a-b| \geq |a|-|b|$ D) $|a-b| = |a|-|b|$

18. … is unbounded set.

A) $\left\{x \mid x=\frac{1}{n}, n \in N\right\}$ C) $\left\{x \mid x=n^2, n \in N\right\}$

B) $\left\{x \mid x=\frac{1}{2^n}, n \in N\right\}$ D) none of these

19. The least upper bound of the set $\left\{\frac{1}{n}, n \in N\right\}$ is …

A) 1 B) 0 C) -1 D) ½

20. Between any two real numbers there exist … rational numbers.

A) No C) finite number of

B) 1 D) infinitely many

21. If M is an upper bound of a set A of real numbers and $M \in A$, then M is…

A) Infimum of A C) supremum of A

B) Both infimum and supremum of A D) none of these

22. Any nonempty subset of real numbers which is bounded below has …

A) supremum C) both infimum and supremum

B) neither infimum nor supremum D) infimum

23. A subset A of real numbers is said to be bounded if it is …

A) Bounded above C) bounded below

B) Both bounded above and bounded below D) none of these

24. … is a countable set.

A) $[0,1]$ B) $[0,1)$ C) $(0,1)$ D) $\left\{0, \pm\frac{1}{n}, n \in N\right\}$

25. Which of the following set is countable?

A) $\{1,4,9,16,25,...\}$ C) all rational numbers

B) $\{2n : n \in N\}$ D) all irrational numbers

26. Point out the wrong statement of the following:

A) The countable union of countable sets is countable.

B) If A and B are countable, then $A \times B$ is countable.

C) The Cartesian product $N \times N$ is uncountable.

D) The set of real numbers is uncountable.

27. If $a \in R$ and P is set of positive real numbers satisfying $a \in P$ or $a = 0$ or $-a \in P$. This property is called …

A) Symmetry property C) trichotomy property

B) Triangular inequality D) none of these

28. If a,b,c be any elements of R and $a > b$ then …

A) $a + c = b + c$ C) $a + c \geq b + c$

B) $a + c < b + c$ D) $a + c > b + c$

29. If $a,b \in R$ and $a > b, c < 0$ then …

A) $ac = bc$ C) $ac > bc$

B) $ac < bc$ D) $ac \leq bc$

30. If $a \in R$ and $a \neq 0$ then …

A) $a^2 > 0$ B) $a^2 = 0$ C) $a^2 < 0$ D) none of these

31. If $a \in R$ such that $0 \leq x < \varepsilon$ for every $\varepsilon > 0$ then …

A) $a = 1$ B) $a > \varepsilon$ C) $a = 0$ D) $a \geq 0$

32. The set A of all real numbers x satisfying $2x + 3 \leq 6$ is …

A) $A = \left\{ x \in R : x = \frac{3}{2} \right\}$ C) $A = \left\{ x \in R : x \leq \frac{3}{2} \right\}$

B) $A = \left\{ x \in R : x \geq \frac{3}{2} \right\}$ D) none of these

33. The set $C = \left\{ x \in R : \frac{2x+1}{x+2} < 1 \right\}$ is …

A) $C = \{ x \in R \mid -1 < x < 1 \}$ C) $C = \{ x \in R \mid 1 < x < 2 \}$

B) $C = \{ x \in R \mid x = 2 \}$ D) $C = \{ x \in R \mid -2 < x < 1 \}$

34. … is the set that satisfy $x^2 > 3x + 4$

A) $x < -1, x > 4$ C) $x = 1$

B) $x > 1, x < -4$ D) $1 \leq x \leq 4$

35. If 'a' is a real number and $|a| = -a$ then …

A) $a = 1$ B) $a < 0$ C) $a > 0$ D) $a < 1$

36. $|a| =$ …. For $a > 0$

A) a B) $-a$ C) $-|a|$ D) none of these

37. $|-8| =$ ….

A) 8 B) -8 C) 1 D) none of these

38. $|0| =$ … and $|1| =$ …, respectively.

A) 1, 0 B) 0, 1 C) 0, 0 D) 1, 1

39. If $a \in R$ then …

A) $|a|^2 = a$ B) $|a|^2 = -a$ C) $|a|^2 = a^2$ D) $|a|^2 = -a^2$

40. Let $a \in R$ and $\varepsilon > 0$. Then ε neighborhood of a is the set …

A) $V_\varepsilon(a)=\{x\in R:|x-a|=0\}$ C) $V_\varepsilon(a)=\{x\in R:|x-a|>\varepsilon\}$

B) $V_\varepsilon(a)=\{x\in R:|x-a|\le\varepsilon\}$ D) $V_\varepsilon(a)=\{x\in R:|x-a|<\varepsilon\}$

41. Let $a\in R$, if x belongs to the neighbor $V_\varepsilon(a)$ for every $\varepsilon>0$ then ...

A) $x=a$ B) $x=\frac{a}{2}$ C) $x=2a$ D) $x=a^2$

42. Every nonempty set of real numbers that has an upper bound also has a … in R.

A) Supremum in R

B) Infimum in R

C) Both infimum and supremum in R

D) None of these

43. Every nonempty set of real numbers that has an upper bound also has a supremum in R is called … property.

A) The completeness property

B) Archimedean property

C) Supremum property

D) None of these

44. If $x\in R$, then there exits $n_x\in N$ such that …

A) $x=n_x$ B) $x>n_x$ C) $x\ge n_x$ D) $x\le n_x$

45. If $x\in R$, then there exits $n_x\in N$ such that $x\le n_x$. This is …

A) The completeness property

B) Archimedean property

C) The density theorem

D) None of these

46. If $S=\left\{\frac{1}{n}:n\in N\right\}$, then inf S = …

A) ∞ B) 1 C) 0 D) 2

47. If x and y are real numbers with $x<y$, then there exists a rational number $r\in Q$ such that $x<r<y$. This is …

A) Archimedean property

B) The supremum property

C) The density theorem

D) The completeness property

48. Sup $\left\{1-\frac{1}{n}: n \in N\right\}$ = …

A) 0 B) 1 C) -1 D) ∞

49. The entire set R ….

A) Is a finite interval

B) Is an infinite interval

C) Has l. u. b.

D) Has g. l. b.

50. If S is subset of R that contains at least two points and has the property, if $x, y \in S$ and $x < y$, then $[x, y] \subseteq S$, then S is an interval. This is …

A) Archimedean property

B) The supremum property

C) The density theorem

D) Characterization theorem

51. The set $C = \{x \in R : |x-1| \lhd |x|\}$ is …

A) $C = \left\{x \in R : x < \frac{1}{2}\right\}$ C) $C = \left\{x \in R : x > \frac{1}{2}\right\}$

B) $C = \left\{x \in R : x = \frac{1}{2}\right\}$ D) $C = \left\{x \in R : x \geq \frac{1}{2}\right\}$

52. … are the values of x that satisfy $3x-1 = |x-7|$.

A) $x \geq 5$ B) $x = 6$ C) $x \leq 6$ D) $5 \leq x < 6$